Bridging Few-Shot Learning and Adaptation: New Challenges of Support-Query Shift

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Abstract. *Few-Shot Learning* (FSL) algorithms have made substantial progress in learning novel concepts with just a handful of labelled data. To classify *query* instances from novel classes encountered at test-time, they only require a *support set* composed of a few labelled samples. FSL benchmarks commonly assume that those queries come from the same distribution as instances in the support set. However, in a realistic setting, data distribution is plausibly subject to change, a situation referred to as *Distribution Shift* (DS). The present work addresses the new and challenging problem of ***F**ew-Shot Learning under **S**upport/**Q**uery **S**hift* (**FSQS**) *i.e.*, when support and query instances are sampled from related but different distributions. Our contributions are the following. First, we release a testbed for FSQS, including datasets, relevant baselines and a protocol for a rigorous and reproducible evaluation. Second, we observe that well-established FSL algorithms unsurprisingly suffer from a considerable drop in accuracy when facing FSQS, stressing the significance of our study. Finally, we show that transductive algorithms can limit the inopportune effect of DS. In particular, we study both the role of Batch-Normalization and Optimal Transport (OT) in aligning distributions, bridging *Unsupervised Domain Adaptation* with FSL. This results in a new method that efficiently combines OT with the celebrated *Prototypical Networks*. We bring compelling experiments demonstrating the advantage of our method. Our work opens an exciting line of research by providing a testbed and strong baselines. Our code is available at https://github.com/ebennequin/meta-domain-shift.

Keywords: Few-Shot Learning · Distribution Shift · Adaptation · Optimal Transport.

1 Introduction

In the last few years, we have witnessed outstanding progress in supervised deep learning [15]. As the abundance of labelled data during training is rarely

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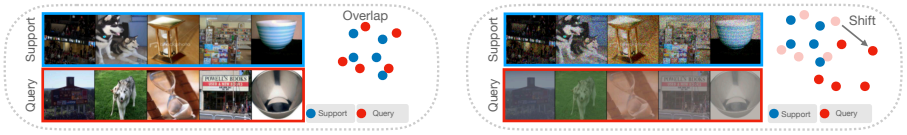

(a) Standard FSL (b) FSL under Support / Query Shift

Fig. 1. Illustration of the FSQS problem with a 5-way 1-shot classification task sampled from the miniImageNet dataset [31]. In (a), a standard FSL setting where support and query sets are sampled from the same distribution. In (b), the same task but with shot-noise and contrast perturbations from [16] applied on support and query sets (respectively) that results in a support-query shift. In the latter case, a similarity measure based on the Euclidean metric [28] may become inadequate.

encountered in practice, ground-breaking works in *Few-Shot Learning* (FSL) have emerged [31, 28, 12], particularly for image classification. This paradigm relies on a straightforward setting. At test-time, given a set of not seen during training and *few* (typically 1 to 5) labelled examples for each of those classes, the task is to classify query samples among them. We usually call the set of labelled samples the *support set*, and the set of query samples the *query set*. Well-adopted FSL benchmarks [31, 25, 30] commonly sample the support and query sets from the same distribution. We stress that this assumption does not hold in most use cases. When deployed in the real-world, we expect an algorithm to infer on data that may shift, resulting in an acquisition system that deteriorates, lighting conditions that vary, or real world objects evolving [1].

The situation of *Distribution Shift* (DS) *i.e.*, when training and testing distributions differ, is ubiquitous and has dramatic effects on deep models [16], motivating works in *Unsupervised Domain Adaptation* [22], *Domain Generalization* [14] or *Test-Time Adaptation* [32]. However, the state of the art brings insufficient knowledge on few-shot learners' behaviours when facing distribution shift. Some pioneering works demonstrate that advanced FSL algorithms do not handle cross-domain generalization better than more naive approaches [5]. Despite its great practical interest, FSL under distribution shift between the support and query sets is an under-investigated problem and attracts a very recent attention [11]. We refer to it as ***F**ew-Shot Learning under **S**upport/**Q**uery **S**hift* (**FSQS**) and provide an illustration in Figure 1. It reflects a more realistic situation where the algorithm is fed with a support set at the time of deployment and infers continuously on data subject to shift. The first solution is to re-acquire a support set that follows the data's evolution. Nevertheless, it implies human intervention to select and annotate data to update an already deployed model, reacting to a potential drop in performances. The second solution consists in designing an algorithm that is robust to the distribution shift encountered during inference. This is the subject of the present work. Our contributions are:

1. `FewShiftBed`: a testbed for FSQS available at `https://github.com/ebennequin/meta-domain-shift`. The testbed includes 3 challenging benchmarks along with a protocol for fair and rigorous comparison across methods as well as

an implementation of relevant baselines, and an interface to facilitate the implementation of new methods.
2. We conduct extensive experimentation of a representative set of few-shot algorithms. We empirically show that *Transductive* Batch-Normalization [3] mitigates an important part of the inopportune effect of FSQS.
3. We bridge *Unsupervised Domain Adaptation* (UDA) with FSL to address FSQS. We introduce *Transported Prototypes*, an efficient transductive algorithm that couples *Optimal Transport* (OT) [23] with the celebrated *Prototypical Networks* [28]. The use of OT follows a long-standing history in UDA for aligning representations between distributions [2, 13]. Our experiments demonstrate that OT shows a remarkable ability to perform this alignment even with only a few samples to compare distributions and provide a simple but strong baseline.

In Section 2 we provide a formal statement of FSQS, and we position this new problem among existing learning paradigms. In Section 3, we present `FewShiftBed`. We detail the datasets, the chosen baselines, and a protocol that guarantees a rigorous and reproducible evaluation. In Section 4, we present a method that couples Optimal Transport with Prototypical Networks [28]. Finally, in Section 5, we conduct an extensive evaluation of baselines and our proposed method using the testbed.

2 The Support-Query Shift problem

2.1 Statement

Notations. We consider an input space $\mathcal{X}$, a representation space $\mathcal{Z} \subset \mathbb{R}^d$ $(d > 0)$ and a set of classes C. A representation is a learnable function from $\mathcal{X}$ to $\mathcal{Z}$ and is noted $\varphi(\cdot;\theta)$ with $\theta \in \Theta$ for Θ a set of parameters. A dataset is a set $\Delta(\mathsf{C},\mathsf{D})$ defined by a set of classes C and a set of domains D *i.e.*, a domain $\mathcal{D} \in \mathsf{D}$ is a set of IID realizations from a distribution noted $p_{\mathcal{D}}$. For two domains $\mathcal{D}, \mathcal{D}' \in \mathsf{D}$, the distribution shift is characterized by $p_{\mathcal{D}} \neq p_{\mathcal{D}'}$. For instance, if the data consists of images of letters handwritten by several users, $\mathcal{D}$ can consist of samples from a specific user. Referring to the well known UDA terminology of source / target [22], we define a couple of source-target domains as a couple $(\mathcal{D}_s, \mathcal{D}_t)$ with $p_{\mathcal{D}_s} \neq p_{\mathcal{D}_t}$, thus presenting a distribution shift. Additionally, given $\mathcal{C} \subset \mathsf{C}$ and $\mathcal{D} \in \mathsf{D}$, the restriction of a domain $\mathcal{D}$ to images with a label that belongs to $\mathcal{C}$ is noted $\mathcal{D}^{\mathcal{C}}$.

Dataset splits. We build a split of $\Delta(\mathsf{C},\mathsf{D})$, by splitting D (respectively C) into $\mathsf{D}_{\text{train}}$ and D_{test} (respectively $\mathsf{C}_{\text{train}}$ and C_{test}) such that $\mathsf{D}_{\text{train}} \cap \mathsf{D}_{\text{test}} = \emptyset$ and $\mathsf{D}_{\text{train}} \cup \mathsf{D}_{\text{test}} = \mathsf{D}$ (respectively $\mathsf{C}_{\text{train}} \cap \mathsf{C}_{\text{test}} = \emptyset$ and $\mathsf{C}_{\text{train}} \cup \mathsf{C}_{\text{test}} = \mathsf{C}$). This gives us a train/test split with the datasets $\Delta_{\text{train}} = \Delta(\mathsf{C}_{\text{train}}, \mathsf{D}_{\text{train}})$ and $\Delta_{\text{test}} = \Delta(\mathsf{C}_{\text{test}}, \mathsf{D}_{\text{test}})$. By extension, we build a validation set following the same protocol.

Fig. 2. During meta-learning (Train-Time), each episode contains a support and a query set sampled from different distributions (for instance, illustrated by noise and contrasts as in Figure 1(b)) from a set of *training domains* ($\mathsf{D}_{\text{train}}$), reflecting a situation that may potentially occurs at test-time. When deployed, the FSL algorithm using a trained backbone is fed with a support set sampled from new classes. As the algorithm is subject to infer continuously on data subject to shift (Test-Time), we evaluate the algorithm on data with an unknown shift (D_{test}). Importantly, both classes ($\mathsf{C}_{\text{train}} \cap \mathsf{C}_{\text{test}} = \emptyset$) and shifts ($\mathsf{D}_{\text{train}} \cap \mathsf{D}_{\text{test}} = \emptyset$) are not seen during training, making the FSQS a challenging problem of generalization.

Few-Shot Learning under Support-Query Shift (FSQS). Given:

- $\mathsf{D}' \in \{\mathsf{D}_{\text{train}}, \mathsf{D}_{\text{test}}\}$ and $\mathsf{C}' \in \{\mathsf{C}_{\text{train}}, \mathsf{C}_{\text{test}}\}$,
- a couple of source-target domains $(\mathcal{D}_s, \mathcal{D}_t)$ from D',
- a set of classes $\mathcal{C} \subset \mathsf{C}'$;
- a small labelled support set $\mathcal{S} = (x_i, y_i)_{i=1,\dots,|\mathcal{S}|}$ (named *source support set*) such that for all i, $y_i \in \mathcal{C}$ and $x_i \in \mathcal{D}_s$ *i.e.*, $\mathcal{S} \subset \mathcal{D}_s^{\mathcal{C}}$;
- an unlabelled query set $\mathcal{Q} = (x_i)_{i=1,\dots,|\mathcal{Q}|}$ (named *target query set*) such that for all i, $y_i \in \mathcal{C}$ and $x_i \in \mathcal{D}_t$ *i.e.*, $\mathcal{Q} \subset \mathcal{D}_t^{\mathcal{C}}$.

The task is to predict the labels of query set instances in $\mathcal{C}$. When $|\mathcal{C}| = n$ and the support set contains k labelled instances for each class, this is called an n-way k-shot FSQS classification task. Note that this paradigm provides an additional challenge compared to classical Few-shot classification tasks, since at test time, the model is expected to generalize to both new classes and new domains while support set and query set are sampled from different distributions. This paradigm is illustrated in Figure 2.

Episodic training. We build an episode by sampling some classes $\mathcal{C} \subset \mathsf{C}_{\text{train}}$, and a source and target domain $\mathcal{D}_s, \mathcal{D}_t$ from $\mathsf{D}_{\text{train}}$. We build a support set $\mathcal{S} = (x_i, y_i)_{i=1\dots|\mathcal{S}|}$ of instances from source domain $\mathcal{D}_s^{\mathcal{C}}$, and a query set $\mathcal{Q} = (x_i, y_i)_{i=|\mathcal{S}|+1,\dots,|\mathcal{S}|+|\mathcal{Q}|}$ of instances from target domain $\mathcal{D}_t^{\mathcal{C}}$, such that $\forall i \in [1, |\mathcal{S}| + |\mathcal{Q}|]$, $y_i \in \mathcal{C}$. Using the labelled examples from $\mathcal{S}$ and unlabelled instances from $\mathcal{Q}$, the model is expected to predict the labels of $\mathcal{Q}$. The parameters of the model are then trained using a cross-entropy loss between the predicted labels and ground truth labels of the query set.

2.2 Positioning and Related Works

To highlight FSQS's novelty, our discussion revolves around the problem of inferring on a given *Query Set* provided with the knowledge of a *Support Set*. We refer to this class of problems as *SQ problems*. Intrinsically, FSL falls into the category of SQ problems. Interestingly, *Unsupervised Domain Adaptation* [22] (UDA), defined as labelling a dataset sampled from a target domain based on labelled data sampled from a source domain, is also a SQ problem. Indeed, in this case, the source domain plays the role of support, while the target domain plays the query's role. Notably, an essential line of study in UDA leverages the target data distribution for aligning source and target domains, reflecting the importance of transduction in a context of adaptation [2, 13] *i.e.*, performing prediction by considering all target samples together. Transductive algorithms also have a special place in FSL [10, 21, 25] and show that leveraging a query set as a whole brings a significant boost in performances. Nevertheless, UDA and FSL exhibit fundamental differences. UDA addresses the problem of distribution shift using important source data and target data (typically thousands of instances) to align distributions. In contrast, FSL focuses on the difficulty of learning from few samples. To this purpose, we frame UDA as both SQ problem with *large* transductivity and Support / Query Shift, while Few-Shot Learning is a SQ problem, eventually with *small* transductivity for transductive FSL. Thus, FSQS combines both challenges: distribution shift and small transductivity. This new perspective allows us to establish fruitful connections with related learning paradigms, presented in Table 1, that we review in the following. A thorough review is available in Appendix A[1].

Adaptation. Unsupervised Domain Adaptation (UDA) requires a whole target dataset for inference, limiting its applications. Recent pioneering works, referred to as Test-Time Adaptation (TTA), adapt at test-time a model provided with a batch of samples from the target distribution. The proposed methodologies are test-time training by self-supervision [29], updating batch-normalization statistics [27] or parameters [32], or meta-learning to condition predictions on the whole batch of test samples for an *Adaptative Risk Minimization* (ARM) [33]. Inspired from the principle of invariant representations [2, 13], the seminal work [7] brings *Optimal Transport* (OT) [23] as an efficient framework for aligning data distributions. OT has been recently applied in a context of transductive FSL [17] and our proposal (TP) is to provide a simple and strong baseline following the principle of OT as it is applied in UDA. In this work, following [3], we also study the role of Batch-Normalization for SQS, that points out the role of transductivity. Our conviction was that the batch-normalization is the first lever for aligning distributions [27, 32].

Few-Shot Classification. We usually frame Few-Shot Classification methods [5] as either metric-based methods [31, 28], or optimization-based methods that learn to fine-tune by adapting with few gradient steps [12]. A promising line of

[1] `https://arxiv.org/abs/2105.11804`

	SQ problems	Train-Time				Test-Time				
		Support		Query		Support		Query	New	New
		Size	Labels	Size	Labels	Size	Labels	Transductivity	classes	domains
No SQS	FSL [28, 12]	Few	✓	Few	✓	Few	✓	Point-wise	✓	✗
No SQS	TransFSL [25, 21]	Few	✓	Few	✓	Few	✓	Small	✓	✗
No SQS	CDFSL [5]	Few	✓	Few	✓	Few	✓	Point-wise	✓	✓
SQS	UDA [24, 22]					Large	✓	Large		
SQS	TTA [29, 27, 32]	Large	✓					Small		✓
SQS	ARM [33]	Large	✓	Few	✓			Small		✓
SQS	Ind FSQS	Few	✓	Few	✓	Few	✓	Point-wise	✓	✓
SQS	Trans FSQS	Few	✓	Few	✓	Few	✓	Small	✓	✓

Table 1. An overview of SQ problems. We divide SQ problems into two categories, presence or not of **S**upport-**Q**uery shift; **No SQS** *vs* **SQS**. We consider three classes of transductivity: point-wise transductivity that is equivalent to inductive inference, small transductivity when inference is performed at batch level (typically in [32, 33]), and large transductivity when inference is performed at dataset level (typically in UDA). New classes (resp. new domains) describe if the model is evaluated at test-time on novel classes (resp. novel domains). Note that we frame UDA as a fully test-time algorithm. Notably, Cross-Domain FSL (CDFSL) [5] assumes that the support set and query set are drawn from the same distribution, thus No SQS.

study leverages *transductivity* (using the query set as unlabelled data while inductive methods predict individually on each query sample). Transductive Propagation Network [21] meta-learns label propagation from the support to query set concurrently with the feature extractor. Transductive Fine-Tuning [10] minimizes the prediction entropy of all query instances during fine-tuning. Evaluating cross-domain generalization of FSL (FSCD), *i.e.*, a distributional shift between meta-training and meta-testing, attracts the attention of a few recent works [5]. Zhao *et al.* propose a Domain-Adversarial Prototypical Network [34] in order to both align source and target domains in the feature space while maintaining discriminativeness between classes. Sahoo *et al.* combine Prototypical Networks with adversarial domain adaptation at the task level [26]. Notably, Cross-Domain Few-Shot Learning [5] (CDFSL) addresses the distributional shift between meta-training and meta-testing assuming that the support set and query set are drawn from the same distribution, not making it a SQ problem with support-query shift. Concerning the novelty of FSQS, we acknowledge the very recent contribution of Du *et al.* [11] which studies the role of learnable normalization for domain generalization, in particular when support and query sets are sampled from different domains. Note that our statement is more ambitious: we evaluate algorithms on both source and target domains that were unseen during training, while in their setting the source domain has already been seen during training.

Benchmarks in Machine Learning Releasing benchmark has always been an important factor for progress in the *Machine Learning* field, the most outstanding example being ImageNet [9] for the Computer Vision community. Recently,

`DomainBed` [14] aims to settle Domain Generalization research into a more rigorous process, where `FewShiftBed` takes inspiration from it. `Meta-Dataset` [30] is an other example, this time specific to FSL.

3 `FewShiftBed`: A `Pytorch` testbed for FSQS

3.1 Datasets

We designed three new image classification datasets adapted to the FSQS problem. These datasets have two specificities.

1. They are dividable into groups of images, assuming that each group corresponds to a distinct domain. A key challenge is that each group must contain enough images with a sufficient variety of class labels, so that it is possible to sample FSQS episodes.
2. They are delivered with a train/val/test split ($\Delta_{\text{train}}, \Delta_{\text{val}}, \Delta_{\text{test}}$), along both the class and the domain axis. This split is performed following the principles detailed in Section 2. Therefore, these datasets provide true few-shot tasks at test time, in the sense that the model will not have seen any instances of test classes and domains during training. Note that since we split along two axes, some data may be discarded (for instance images from a domain in $\mathsf{D}_{\text{train}}$ with a label in C_{test}). Therefore it is crucial to find a split that minimizes this loss of data.

Meta-CIFAR100-Corrupted (MC100-C). CIFAR-100 [19] is a dataset of 60k three-channel square images of size 32×32, evenly distributed in 100 classes. Classes are evenly distributed in 20 superclasses. We use the same method used to build CIFAR-10-C [16], which makes use of 19 image perturbations, each one being applied with 5 different levels of intensity, to evaluate the robustness of a model to domain shift. We modify their protocol to adapt it to the FSQS problem: (i) we split the classes with respect to the superclass structure, and assign 13 superclasses (65 classes) to the training set, 2 superclasses (10 classes) to the validation set, and 5 superclasses (25 classes) to the testing set; (ii) we also split image perturbations (acting as domains), following the split of [33]. We obtain 2,184k transformed images for training, 114k for validation and 330k for testing. The detailed split is available in the documentation of our code repository.

*miniImageNet-Corrupted (mIN-C). mini*ImageNet [31] is a popular benchmark for few-shot image classification. It contains 60k images from 100 classes from the ImageNet dataset. 64 classes are assigned to the training set, 16 to the validation set and 20 to the test set. Like MC100-C, we build mIN-C using the image perturbations proposed by [16] to simulate different domains. We use the original split from [31] for classes, and use the same domain split as for MC100-C. Although the original *mini*ImageNet uses 84×84 images, we use 224×224 images. This allows us to re-use the perturbation parameters calibrated in [16] for ImageNet. Finally, we discard the 5 most time-consuming perturbations. We obtain a total of 1.2M transformed images for training, 182k for validation and 228k for testing. The detailed split in the documentation of our code repository.

FEMNIST-FewShot (FEMNIST-FS). EMNIST [6] is a dataset of images of handwritten digits and uppercase and lowercase characters. Federated-EMNIST [4] is a version of EMNIST where images are sorted by writer (or user). FEMNIST-FS consists in a split of the FEMNIST dataset adapted to few-shot classification. We separate both users and classes between training, validation and test sets. We build each group as the set of images written by one user. The detailed split is available in the code. Note that in FEMNIST, many users provide several instances for each digits, but less than two instance for most letters. Therefore it is hard to find enough samples from a user to build a support set or a query set. As a result, our experiments are limited to classification tasks with only one sample per class in both the support and query sets.

3.2 Algorithms

We implement in `FewShiftBed` two representative methods of the vast literature of FSL, that are commonly considered as strong baselines: Prototypical Networks (**ProtoNet**) [28] and Matching Networks (**MatchingNet**) [31]. Besides, for transductive FSL, we also implement with Transductive Propagation Network (**TransPropNet**) [21] and Transductive Fine-Tuning (**FTNet**) [10]. We also implement our novel algorithm *Transported Prototypes* (**TP**) which is detailed in Section 4. `FewShiftBed` is designed for favoring a straightforward implementation of a new algorithm for FSQS. To add a new algorithm, we only need to implement the `set_forward` method of the class `AbstractMetaLearner`. We provide an example with our implementation of the Prototypical Network [28] that only requires few line of codes:

```
class ProtoNet(AbstractMetaLearner):
    def set_forward(self, support_images, support_labels, query_images):
        z_support, z_query = self.extract_features(support_images, query_images)
        z_proto = self.get_prototypes(z_support, support_labels)
        return  - euclidean_dist(z_query, z_proto)
```

3.3 Protocol

To prevent the pitfall of misinterpreting a performance boost, we draw three recommendations to isolate the causes of improvement rigorously.

- **How important is episodic training?** Despite its wide adoption in meta-learning for FSL, in some situation episodic training does not perform better than more naive approaches [5]. Therefore we recommend to report both the result obtained using episodic training and standard ERM (see the documentation of our code repository).
- **How does the algorithm behave in the absence of Support-Query Shift?** In order to assess that an algorithm designed for distribution shift does not provide degraded performance in an ordinary concept, and to provide a top-performing baseline, we recommend reporting the model's performance when we do not observe, at test-time, a support-query shift. Note that it is equivalent to evaluate the performance in cross-domain generalization, as firstly described in [5].

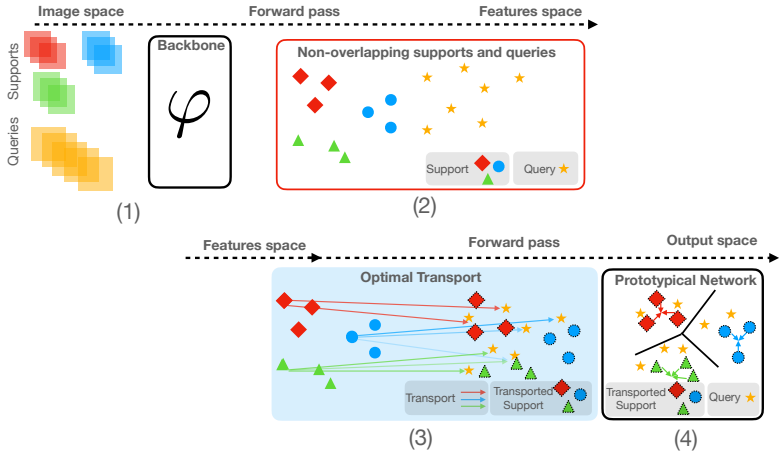

Fig. 3. Overview of *Transported Prototypes*. (1) A support set and a query set are fed to a trained backbone that embeds images into a feature space. (2) Due to the shift between distributions, support and query instances are embedded in non-overlapping areas. (3) We compute the Optimal Transport from support instances to query instances to build the transported support set. Note that we represent the transport plan only for one instance per class to preserve clarity in the schema. (4) Provided with the transported support, we apply the Prototypical Network [28] *i.e.*, L^2 similarity between transported support and query instances.

– **Is the algorithm transductive?** The assumption of transductivity has been responsible of several improvements in FSL [25, 3] while it has been demonstrated in [3] that MAML [12] benefits strongly from the Transductive Batch-Normalization (TBN). Thus, we recommend specifying if the method is transductive and adapting the choice of the batch-normalization accordingly (Conventional Batch Normalization [18] and Transductive Batch Normalization for inductive and transductive methods, respectively) since transductive batch normalization brings a significant boost in performance [3].

4 Transported Prototypes: A baseline for FSQS

4.1 Overall idea

We present a novel method that brings UDA to FSQS. As aforementioned, FSQS presents new challenges since we no longer assume that we sample the support set and the query set from the same distribution. As a result, it is unlikely that the support set and query sets share the same representation space region (non-overlap). In particular, the L^2 distance, adopted in the celebrated Prototypical Network [28], may not be relevant for measuring similarity between query and support instances, as presented in Figure 1. To overcome this issue, we develop a two-phase approach that combines Optimal Transport (Transportation Phase)

and the celebrated Prototypical Network (Prototype Phase). We give some background about Optimal Transport (OT) in Section 4.2 and the whole procedure is presented in Algorithm 1.

4.2 Background

Definition. We provide some basics about Optimal Transport (OT). A thorough presentation of OT is available at [23]. Let p_s and p_t be two distributions on $\mathcal{X}$, we note $\Pi(p_s, p_t)$ the set of joint probability with marginal p_s and p_t *i.e.*, $\forall \pi \in \Pi(p_s, p_t), \forall x \in \mathcal{X}, \pi(\cdot, x) = p_s, \pi(x, \cdot) = p_t$. The *Optimal Transport*, associated to cost c, between p_s and p_t is defined as:

$$W_c(p_s, p_t) := \min_{\pi \in \Pi(p_s, p_t)} \mathbb{E}_{(x_s, x_t) \sim \pi} \left[c(x_s, x_t) \right] \tag{1}$$

with $c(\cdot, \cdot)$ any metric. We note $\pi^\star(p_s, p_t)$ the joint distribution that achieves the minimum in equation 1. It is named the *transportation plan* from p_s to p_t. When there is no confusion, we simply note $\pi^\star$. For our applications, we will use as metric the euclidean distance in the representation space obtained from a representation $\varphi(\cdot; \theta)$ *i.e.*, $c_\theta(x_s, x_t) := ||\varphi(x_s; \theta) - \varphi(x_t; \theta)||_2$.

Discrete OT. When p_s and p_t are only accessible through a finite set of samples, respectively $(x_{s,1}, ..., x_{s,n_s})$ and $(x_{t,1}, ..., x_{t,n_t})$ we introduce the empirical distributions $\hat{p}_s := \sum_{i=1}^{n_s} w_{s,i} \delta_{x_{s,i}}$, $\hat{p}_t := \sum_{j=1}^{n_t} w_{t,j} \delta_{x_{t,j}}$, where $w_{s,i}$ ($w_{t,j}$) is the mass probability put in sample $x_{s,i}$ ($x_{t,j}$) *i.e.*, $\sum_{i=1}^{n_s} w_{s,i} = 1$ ($\sum_{j=1}^{n_t} w_{t,j} = 1$) and δ_x is the Dirac distribution in x. The discrete version of the OT is derived by introducing the set of couplings $\mathbf{\Pi}(p_s, p_t) := \left\{ \boldsymbol{\pi} \in \mathbb{R}^{n_s \times n_t}, \boldsymbol{\pi} \mathbf{1}_{n_s} = \mathbf{p}_s, \boldsymbol{\pi}^\top \mathbf{1}_{n_t} = \mathbf{p}_t \right\}$ where $\mathbf{p}_s := (w_{s,1}, \cdots, w_{s,n_s})$, $\mathbf{p}_t := (w_{t,1}, \cdots, w_{1,n_t})$, and $\mathbf{1}_{n_s}$ (respectively $\mathbf{1}_{n_t}$) is the unit vector with dim n_s (respectively n_t). The discrete transportation plan $\boldsymbol{\pi}_\theta^\star$ is then defined as:

$$\boldsymbol{\pi}_\theta^\star := \underset{\boldsymbol{\pi} \in \mathbf{\Pi}(p_s, p_t)}{\operatorname{argmin}} \langle \boldsymbol{\pi}, \mathbf{C}_\theta \rangle_F \tag{2}$$

where $\mathbf{C}_\theta(i, j) := c_\theta(x_{s,i}, x_{t,j})$ and $\langle \cdot, \cdot \rangle_F$ is the Frobenius dot product. Note that $\boldsymbol{\pi}_\theta^\star$ depends on both p_s and p_t, and θ since $\mathbf{C}_\theta$ depends on θ. In practice, we use Entropic regularization [8] that makes OT easier to solve by promoting smoother transportation plan with a computationally efficient algorithm, based on Sinkhorn-Knopp's scaling matrix approach (see the Appendix C).

4.3 Method

Transportation Phase. At each episode, we are provided with a source support set $\mathcal{S}$ and a target query set $\mathcal{Q}$. We note respectively $\mathbf{S}$ and $\mathbf{Q}$ their representations from a deep network $\varphi(\cdot; \theta)$ *i.e.*, $z_s \in \mathbf{S}$ is defined as $z_s := \varphi(x_s; \theta)$ for $x_s \in \mathcal{S}$, respectively $z_q \in \mathbf{Q}$ is defined as $z_q := \varphi(x_q; \theta)$ for $x_q \in \mathcal{Q}$. As these two sets are sampled from different distributions, $\mathbf{S}$ and $\mathbf{Q}$ are likely to lie in different regions of the representation space. In order to adapt the source support set $\mathcal{S}$

Algorithm 1 Transported Prototypes. Blue lines highlight the OT's contribution in the computational graph of an episode compared to the standard Prototypical Network [28].

Input: Support set $\mathcal{S} := (x_{s,i}, y_{s,i})_{1 \leq i \leq n_s}$, query set $\mathcal{Q} := (x_{q,j}, y_{q,j})_{1 \leq j \leq n_q}$, classes $\mathcal{C}$, backbone φ_θ.
Output: Loss $\mathcal{L}(\theta)$ for a randomly sampled episode.

$z_{s,i}, z_{q,j} \leftarrow \varphi(x_{s,i}; \theta), \varphi(x_{q,j}; \theta)$, for i, j ▷ Get representations.
$\mathbf{C}_\theta(i,j) \leftarrow ||z_{s,i} - z_{q,j}||^2$, for i, j ▷ Cost-matrix.
$\boldsymbol{\pi}^\star_\theta \leftarrow$ Solve Equation 2 ▷ Transportation plan.
$\hat{\boldsymbol{\pi}}^\star_\theta(i,j) \leftarrow \boldsymbol{\pi}^\star_\theta(i,j) / \sum_j \boldsymbol{\pi}^\star_\theta(i,j)$, for i, j ▷ Normalization.
$\hat{\mathbf{S}} = (\hat{z}_{s,i})_i \leftarrow$ Given by Equation 3 ▷ Get transported support set.
$\hat{\mathbf{c}}_k \leftarrow \frac{1}{|\hat{\mathbf{S}}_k|} \sum_{\hat{z}_s \in \hat{\mathbf{S}}_k} \hat{z}_s$, for $k \in \mathcal{C}$. ▷ Get transported prototypes.
$p_\theta(y|x_{q,j}) \leftarrow$ From Equation 4, for j
Return: $\mathcal{L}(\theta) := \frac{1}{n_q} \sum_{j=1}^{n_q} -\log p_\theta(y_{q,j}|x_{q,j})$.

to the target domain, which is only represented by the target query set $\mathcal{Q}$, we follow [7] to compute $\hat{\mathbf{S}}$ the *barycenter mapping* of $\mathcal{S}$, that we refer to as the *transported support set*, defined as follows:

$$\hat{\mathbf{S}} := \hat{\boldsymbol{\pi}}^\star_\theta \mathbf{Q} \tag{3}$$

where $\boldsymbol{\pi}^\star_\theta$ is the transportation plan from $\mathbf{S}$ to $\mathbf{Q}$ and $\hat{\boldsymbol{\pi}}^\star_\theta := \boldsymbol{\pi}^\star_\theta(i,j) / \sum_{j=1}^{n_t} \boldsymbol{\pi}^\star_\theta(i,j)$. The *transported* support set $\hat{\mathbf{S}}$ is an estimation of labelled examples in the target domain using labelled examples in the source domain. The success relies on the fact that transportation conserves labels, *i.e.*, a query instance close to $\hat{z}_s \in \hat{\mathbf{S}}$ should share the same label with x_s, where $\hat{z}_s$ is the barycenter mapping of $z_s \in \mathbf{S}$. See step (3) of Figure 3 for a visualization of the transportation phase.

Prototype Phase. For each class $k \in \mathcal{C}$, we compute the *transported prototypes* $\hat{\mathbf{c}}_k := \frac{1}{|\hat{\mathbf{S}}_k|} \sum_{\hat{z}_s \in \hat{\mathbf{S}}_k} \hat{z}_s$ (where $\hat{\mathbf{S}}_k$ is the transported support set with class k and $\mathcal{C}$ are classes of current episode). We classify each query x_q with representation $z_q = \varphi(x_q; \theta)$ using its euclidean distance to each transported prototypes;

$$p_\theta(y = k|x_q) := \frac{\exp\left(-||z_q - \hat{\mathbf{c}}_k||^2\right)}{\sum_{k' \in \mathcal{C}} \exp\left(-||z_q - \hat{\mathbf{c}}_{k'}||^2\right)} \tag{4}$$

Crucially, the standard Prototypical Networks [28] computes euclidean distance to each prototypes while we compute the euclidean to each *transported* prototypes, as presented in step (4) of Figure 3. Note that our formulation involves the query set in the computation of $(\hat{\mathbf{c}}_k)_{k \in \mathcal{C}}$.

Genericity of OT. `FewShiftBed` implements OT as a stand-alone module that can be easily plugged into any FSL algorithm. We report additional baselines in Appendix B where other FSL algorithms are equipped with OT. This technical choice reflects our insight that OT may be ubiquitous for addressing FSQS and makes its usage in the testbed straightforward.

	Meta-CIFAR100-C		miniImageNet-C		FEMNIST-FS
	1-shot	**5-shot**	**1-shot**	**5-shot**	**1-shot**
ProtoNet [28]	30.02 ± 0.40	42.77 ± 0.47	36.37 ± 0.50	47.58 ± 0.57	84.31 ± 0.73
MatchingNet [31]	30.71 ± 0.38	41.15 ± 0.45	35.26 ± 0.50	44.75 ± 0.55	84.25 ± 0.71
TransPropNet† [21]	**34.15 ± 0.39**	47.39 ± 0.42	24.10 ± 0.27	27.24 ± 0.33	86.42 ± 0.76
FTNet† [10]	28.91 ± 0.37	37.28 ± 0.40	39.02 ± 0.46	51.27 ± 0.45	86.13 ± 0.71
TP† (ours)	**34.00 ± 0.46**	**49.71 ± 0.47**	**40.49 ± 0.54**	**59.85 ± 0.49**	**93.63 ± 0.63**
TP w/o OT †	32.47 ± 0.41	48.00 ± 0.44	40.43 ± 0.49	53.71 ± 0.50	90.36 ± 0.58
TP w/o TBN †	33.74 ± 0.46	49.18 ± 0.49	37.32 ± 0.55	55.16 ± 0.54	92.31 ± 0.73
TP w. OT-TT †	32.81 ± 0.46	48.62 ± 0.48	**44.77 ± 0.57**	**60.46 ± 0.49**	**94.92 ± 0.55**
TP w/o ET †	**35.94 ± 0.45**	48.66 ± 0.46	42.46 ± 0.53	54.67 ± 0.48	94.22± 0.70
TP w/o SQS †	*85.67 ± 0.26*	*88.52 ± 0.17*	*64.27 ± 0.39*	*75.22 ± 0.30*	*92.65 ± 0.69*

Table 2. Top-1 accuracy of few-shot learning models in various datasets and numbers of shots with 8 instances per class in the query set (except for FEMNIST-FS: 1 instance per class in the query set), with 95% confidence intervals. The top half of the table is a comparison between existing few-shot learning methods and Transported Prototypes (TP). The bottom half is an ablation study of TP. OT denotes Optimal Transport, TBN is Transductive Batch-Normalization, OT-TT refers to the setting where Optimal Transport is applied at test time but not during episodic training, and ET means episodic training *i.e.*, w/o ET refers to the setting where training is performed through standard Empirical Risk Minimization. TP w/o SQS reports model's performance in the absence of support-query shift. † flags if the method is transductive. For each setting, the best accuracy among existing methods is shown in bold, as well as the accuracy of an ablation if it improves TP.

5 Experiments

We compare the performance of baseline algorithms with *Transported Prototypes* on various datasets and settings. We also offer an ablation study in order to isolate the source to the success of *Transported Prototypes*. Extensive results are detailed in Appendix B. Instructions to reproduce these results can be found in the code's documentation.

Setting and details. We conduct experiments on all methods and datasets implemented in `FewShiftBed`. We use a standard 4-layer convolutional network for our experiments on Meta-CIFAR100-C and FEMNIST-FewShot, and a ResNet18 for our experiments on miniImageNet. Transductive methods are equipped with a Transductive Batch-Normalization. All episodic training runs contain 40k episodes, after which we retrieve model state with best validation accuracy. We run each individual experiment on three different random seeds. All results presented in this paper are the average accuracies obtained with these random seeds.

Analysis. The top half of Table 2 reveals that Transported Prototypes (TP) outperform all baselines by a strong margin on all datasets and settings. Importantly, baselines perform poorly on FSQS, demonstrating they are not equipped to address this challenging problem, stressing our study's significance. It is also

interesting to note that the performance of transductive approaches, which is significantly better in a standard FSL setting [21, 10], is here similar to inductive methods (notably, TransPropNet [21] fails loudly without Transductive Batch-Normalization showing that propagating label with non-overlapping support/query can have a dramatic impact, see Appendix B). Thus, FSQS deserves a fresher look to be solved. Transported Prototypes mitigate a significant part of the performance drop caused by support-query shift while benefiting from the simplicity of combining a popular FSL method with a time-tested UDA method. This gives us strong hopes for future works in this direction.

Ablation study. Transported Prototypes (TP) combines three components: Optimal Transport (OT), Transductive Batch-Normalization (TBN) and episode training (ET). Which of these components are responsible for the observed gain? Following recommendations from Section 3.3, we ablate those components in the bottom half of Table 2. We observe that both OT and TBN individually improve the performance of ProtoNet for FSQS, and that the best results are obtained when the two of them are combined. Importantly, OT without TBN performs better than TBN without OT (except for 1-shot mIN-C), demonstrating the superiority of OT compared to TBN for aligning distributions in the few samples regime. Note that the use of TaskNorm [3] is beyond the scope of the paper[2]; we encourage future work to dig into that direction and we refer the reader to the very recent work [11]. We observe that there is no clear evidence that using OT at train-time is better than simply applying it at test-time on a ProtoNet trained without OT. Additionally, the value of Episodic Training (ET) compared to standard Empirical Risk Minimization (ERM) is not obvious. For instance, simply training with ERM and applying TP at test-time is better than adding ET on 1-shot MC100-C, 1-shot mIN-C and FEMNIST-FS, making it an another element to add to the study [20] who put into question the value of ET. Understanding why and when we should use ET or only OT at test-time is interesting for future works. Additionally, we compare TP with MAP [17] which implements an OT-based approach for transductive FSL. Their approach includes a power transform to reduce the skew in the distribution, so for fair comparison we also implemented it into Transported Prototypes for these experiments[3]. We also used the OT module only at test-time and compared with two backbones, respectively trained with ET and ERM. Interestingly, our experiments in Table 3 show that MAP is able to handle SQS. Finally, in order to evaluate the performance drop related to Support-Query Shift compared to a setting with support and query instances sampled from the same distribution, we test Transported Prototypes on few-shot classification tasks without SQS (TP w/o SQS in Table 2), making a setup equivalent to CDFSL. Note that in both cases, the model is trained in an episodic fashion on tasks presenting a Support-Query Shift. These results show that SQS presents a significantly harder challenge than CDFSL, while there is considerable room for improvements.

[2] These normalizations are implemented in `FewShiftBed` for future works.
[3] Therefore results in Table 3 differ from results in Table 2.

	Meta-CIFAR100-C		miniImageNet-C		FEMNIST-FS
	1-shot	**5-shot**	**1-shot**	**5-shot**	**1-shot**
TP*	36.17 ± 0.47	50.45 ± 0.47	45.41 ± 0.54	57.82 ± 0.48	93.60 ± 0.68
MAP*	35.96 ± 0.44	49.55 ± 0.45	43.51 ± 0.47	56.10 ± 0.43	92.86 ± 0.67
TP†	32.13 ± 0.45	46.19 ± 0.47	45.77 ± 0.58	59.91 ± 0.48	94.92 ± 0.56
MAP†	32.38 ± 0.41	45.96 ± 0.43	43.81 ± 0.47	57.70 ± 0.43	87.15 ± 0.66

Table 3. Top-1 accuracy with 8 instances per class in the query set when applying Transported Prototypes and MAP on two different backbones: $\star$ is standard ERM (*i.e.*, without Episodic Training) and † is ProtoNet [28]. Transported Prototypes performs equally or better than MAP [17]. Here TP includes power transform in the feature space.

6 Conclusion

We release `FewShiftBed`, a testbed for the under-investigated and crucial problem of Few-Shot Learning when the support and query sets are sampled from related but different distributions, named FSQS. `FewShiftBed` includes three datasets, relevant baselines and a protocol for reproducible research. Inspired from recent progress of Optimal Transport (OT) to address Unsupervised Domain Adaptation, we propose a method that efficiently combines OT with the celebrated Prototypical Network [28]. Following the protocol of `FewShiftBed`, we bring compelling experiments demonstrating the advantage of our proposal compared to transductive counterparts. We also isolate factors responsible for improvements. Our findings suggest that Batch-Normalization is ubiquitous, as described in related works [3, 11], while episodic training, even if promising on paper, is questionable. As a lead for future works, `FewShiftBed` could be improved by using different datasets to model different domains, instead of using artificial transformations. Since we are talking about domain adaptation, we also encourage the study of accuracy as a function of the size of the target domain, *i.e.*, the size of the query set. Moving beyond the transductive algorithm, as well as understanding when meta-learning brings a clear advantage to address FSQS remains an open and exciting problem. `FewShiftBed` brings the first step towards its progress.

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References

[1] Dario Amodei et al. "Concrete problems in AI safety". In: *arXiv preprint arXiv:1606.06565* (2016).

[2] Shai Ben-David et al. "Analysis of representations for domain adaptation". In: *Advances in neural information processing systems*. 2007, pp. 137–144.

[3] John Bronskill et al. "Tasknorm: Rethinking batch normalization for meta-learning". In: *ICML*. PMLR. 2020, pp. 1153–1164.

[4] Sebastian Caldas et al. "Leaf: A benchmark for federated settings". In: *arXiv preprint arXiv:1812.01097* (2018).

[5] Wei-Yu Chen et al. "A Closer Look at Few-shot Classification". In: *International Conference on Learning Representations*. 2019.

[6] Gregory Cohen et al. "EMNIST: Extending MNIST to handwritten letters". In: *IJCNN*. IEEE. 2017.

[7] Nicolas Courty et al. "Optimal transport for domain adaptation". In: *IEEE transactions on pattern analysis and machine intelligence* 39.9 (2016), pp. 1853–1865.

[8] Marco Cuturi. "Sinkhorn distances: Lightspeed computation of optimal transport". In: *Advances in neural information processing systems* 26 (2013), pp. 2292–2300.

[9] Jia Deng et al. "Imagenet: A large-scale hierarchical image database". In: *2009 IEEE conference on computer vision and pattern recognition*. Ieee. 2009, pp. 248–255.

[10] Guneet Singh Dhillon et al. "A Baseline for Few-Shot Image Classification". In: *ICLR*. 2020.

[11] Yingjun Du et al. "MetaNorm: Learning to Normalize Few-Shot Batches Across Domains". In: *International Conference on Learning Representations*. 2021. URL: https://openreview.net/forum?id=9z_dNsC4B5t.

[12] Chelsea Finn et al. "Model-agnostic meta-learning for fast adaptation of deep networks". In: *ICML*. JMLR. 2017.

[13] Yaroslav Ganin and Victor Lempitsky. "Unsupervised Domain Adaptation by Backpropagation". In: *International Conference on Machine Learning*. 2015, pp. 1180–1189.

[14] Ishaan Gulrajani and David Lopez-Paz. "In Search of Lost Domain Generalization". In: *International Conference on Learning Representations*. 2021.

[15] Kaiming He et al. "Deep residual learning for image recognition". In: *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2016, pp. 770–778.

[16] Dan Hendrycks and Thomas Dietterich. "Benchmarking Neural Network Robustness to Common Corruptions and Perturbations". In: *ICLR*. 2019.

[17] Yuqing Hu et al. "Leveraging the feature distribution in transfer-based few-shot learning". In: *arXiv preprint arXiv:2006.03806* (2020).

[18] Sergey Ioffe and Christian Szegedy. "Batch normalization: Accelerating deep network training by reducing internal covariate shift". In: *ICML*. PMLR. 2015.

[19] Alex Krizhevsky et al. "Learning multiple layers of features from tiny images". In: *Citeseer* (2009).
[20] Steinar Laenen and Luca Bertinetto. "On Episodes, Prototypical Networks, and Few-shot Learning". In: *arXiv preprint arXiv:2012.09831* (2020).
[21] Yanbin Liu et al. "Learning to propagate labels: Transductive propagation network for few-shot learning". In: *ICLR* (2019).
[22] Sinno Jialin Pan and Qiang Yang. "A survey on transfer learning". In: *IEEE Transactions on knowledge and data engineering* 22.10 (2009), pp. 1345–1359.
[23] Gabriel Peyré et al. "Computational Optimal Transport: With Applications to Data Science". In: *Foundations and Trends® in Machine Learning* 11.5-6 (2019), pp. 355–607.
[24] Joaquin Quionero-Candela et al. *Dataset shift in machine learning*. The MIT Press, 2009.
[25] Mengye Ren et al. "Meta-learning for semi-supervised few-shot classification". In: *ICLR* (2019).
[26] Doyen Sahoo et al. *Meta-Learning with Domain Adaptation for Few-Shot Learning under Domain Shift*. 2019.
[27] Steffen Schneider et al. "Improving robustness against common corruptions by covariate shift adaptation". In: *Advances in Neural Information Processing Systems* 33 (2020).
[28] Jake Snell et al. "Prototypical networks for few-shot learning". In: *Advances in Neural Information Processing Systems*. 2017, pp. 4077–4087.
[29] Yu Sun et al. "Test-time training with self-supervision for generalization under distribution shifts". In: *ICML*. 2020.
[30] Eleni Triantafillou et al. "Meta-dataset: A dataset of datasets for learning to learn from few examples". In: *ICLR* (2020).
[31] Oriol Vinyals et al. "Matching networks for one shot learning". In: *NIPS*. 2016.
[32] Dequan Wang et al. "Fully test-time adaptation by entropy minimization". In: *ICLR* (2021).
[33] Marvin Zhang et al. "Adaptive Risk Minimization: A Meta-Learning Approach for Tackling Group Shift". In: *ICLR* (2021).
[34] An Zhao et al. "Domain-Adaptive Few-Shot Learning". In: *arXiv preprint arXiv:2003.08626* (2020).